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Massive higher-dimensional gauge fields as messengers of supersymmetry breaking

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Abstract: We consider theories with one or more compact dimensions with size $r > 1/M$, where M is the fundamental Planck scale, with the visible and hidden sectors localized on spatially separated '3-branes.' We show that a bulk U(1) gauge field spontaneously broken on the hidden-sector 3-brane is an attractive candidate for the messenger of supersymmetry breaking. In this scenario scalar mass-squared terms are proportional to U(1) charges, and therefore naturally conserve flavor. Arbitrary flavor violation at the Planck scale gives rise to exponentially suppressed flavor violation at low energies. Gaugino masses can be generated if the standard gauge fields propagate in the bulk; μ and $B\mu$ terms can be generated by the Giudice-Masiero or by the VEV of a singlet in the visible sector. The latter case naturally solves the SUSY CP problem. Realistic phenomenology can be obtained either if all microscopic parameters are order one in units of M, or if the theory is strongly coupled at the scale M. In either case, the only unexplained hierarchy is the 'large' size of the extra dimensions in fundamental units, which need only be an order of magnitude. All soft masses are naturally within an order of magnitude of $m_{3/2}$, and trilinear scalar couplings are negligible. Squark and slepton masses can naturally unify even in the absence of grand unification.

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Contents

1. Introduction

The hidden-sector scenario for supersymmetry (SUSY) breaking [1] is arguably the simplest and most natural mechanism for realizing SUSY in nature. In this scenario, one assumes that there is a hidden sector in which SUSY is broken, and that Planck-scale suppressed interactions arising from supergravity (SUGRA) or string theory couple the hidden and visible sectors. Scalar masses are assumed to arise from higher-dimension operators of the form

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^4\theta \, \frac{1}{M^2} (\Sigma^\dagger \Sigma)(Q^\dagger Q) \,, \tag{1.1}$$

where Q is an visible sector field, and Σ is a hidden sector field with $\langle F_\Sigma \rangle \neq 0$. Gaugino masses can arise from

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^2\theta \, \frac{1}{M} \Sigma W^\alpha W_\alpha + \text{h.c.} \,, \tag{1.2}$$

and μ and $B\mu$ terms can arise from the Giudice-Masiero term [2]

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^4\theta \left[\frac{1}{M} \Sigma^\dagger + \frac{1}{M^2} \Sigma^\dagger \Sigma \right] H_u H_d + \text{h.c.} \tag{1.3}$$

In addition, trilinear scalar interactions (A terms) can arise from operators of the form

$$\Delta\mathcal{L}_{\rm eff} \sim \int d^2\theta \, \frac{1}{M} \Sigma Q Q H_{u,d} + \text{h.c.} \tag{1.4}$$

If M is of order the Planck scale and $\langle F_\Sigma \rangle \sim (10^{11}\,\mathrm{GeV})^2$, this naturally generates all required soft SUSY breaking terms at the weak scale with $m_{\mathrm{soft}} \sim m_{3/2}$. This scenario is attractive from a theoretical point of view because all of the ingredients are either there of necessity (e.g. supergravity) or arise naturally (e.g. hidden sectors are a generic consequence of string theory). In order to obtain a gaugino mass (and μ, $B\mu$ and A terms) in this way, Σ must be a gauge singlet, so the hidden sector must contain a singlet with a large F term.[1]

The major difficulty with this scenario is that there is no compelling reason for the interactions eq. (1.1) that communicate between the hidden and visible sector to preserve flavor. Off-diagonal squark masses are severely constrained by FCNC's; for example, mixing and CP violation in the K system give[2]

$$\frac{m^2_{\tilde{d}\tilde{s}}}{m^2_{\tilde{s}}} \lesssim (6\times 10^{-3})\left(\frac{m_{\tilde{s}}}{1\,\mathrm{TeV}}\right), \qquad \mathrm{Im}\left(\frac{m^2_{\tilde{d}\tilde{s}}}{m^2_{\tilde{s}}}\right) \lesssim (4\times 10^{-4})\left(\frac{m_{\tilde{s}}}{1\,\mathrm{TeV}}\right). \tag{1.5}$$

An elegant solution to this problem was proposed by Randall and Sundrum in ref. [5]. They considered a situation where the hidden and visible sectors are localized on spatially separated '3-branes' in $D > 4$ spacetime dimensions, with only supergravity propagating in the bulk. (This is similar to the Hořava-Witten vacuum in the context of M theory [6].) Ref. [5] pointed out that in this set-up flavor-violating interactions between the hidden and visible sectors from short-distance physics are suppressed even if the underlying theory does not conserve flavor. The reason is that the exchange of particles with masses of order the D-dimensional Planck scale M_D (e.g. massive string states) is exponentially suppressed by Yukawa factors $\sim e^{-M_D r}$, where r is the distance between the sectors. A modest hierarchy $r \gtrsim 10/M_D$ is therefore sufficient to suppress flavor-changing neutral currents.

If only supergravity propagates in the bulk, the leading contribution to soft masses is directly related to the conformal anomaly [5, 7], and gives calculable scalar and gaugino masses proportional to anomalous dimensions. (It is nontrivial that exchange of supergravity KK modes does not give rise to contact interactions of the form eq. (1.1). This is discussed in detail in ref. [8].) This 'anomaly mediated' scenario is attractive in that it automatically gives flavor-diagonal scalar masses, but it suffers from a number of drawbacks. Most importantly, the slepton mass-squared terms are negative in the MSSM. Also, the Giudice-Masiero mechanism does not naturally solve the μ problem. There have been a number of proposals to make this scenario realistic without spoiling its attractive features [9, 10, 11].

In this paper, we consider a variation on this scenario that naturally conserves flavor while preserving the desirable features of hidden sector models described above. Following ref. [5], we consider models where SUSY is broken on a spatially separated

[1] Dynamical SUSY breaking models with this feature are discussed in refs. [3].

[2] For a complete discussion, see e.g. ref. [4].

3-brane. This guarantees the absence of FCNC's from uncalculable contact interactions of the form eq. (1.1). The new ingredient we add is a $\mathrm{U}(1)_X$ gauge multiplet that propagates in the bulk and couples the fields in the hidden and visible sectors. $\mathrm{U}(1)_X$ is assumed to be spontaneously broken by vacuum expectation values of charged scalars localized on the hidden-sector 3-brane. Exchange of the massive $\mathrm{U}(1)_X$ gauge boson gives rise to terms in the 4-dimensional effective theory of the form

$$\Delta\mathcal{L}_{4,\mathrm{eff}} \sim \int d^4\theta\, \frac{1}{v^2}(\Sigma^\dagger X \Sigma)(Q^\dagger X Q)\,, \tag{1.6}$$

where v is the VEV that breaks $\mathrm{U}(1)_X$ and X is the charge matrix. If $\langle F_\Sigma \rangle \neq 0$, this gives rise to visible sector scalar mass-squared terms proportional to their $\mathrm{U}(1)_X$ charges. It is natural for all fields with the same standard-model.

If $\mathrm{U}(1)_X$ commutes with flavor symmetries, then all fermions with the same gauge quantum numbers will have the same scalar mass-squared, which does not give FCNC's.

The SUSY breaking terms induced by $\mathrm{U}(1)_X$ exchange preserve $\mathrm{U}(1)_R$, and therefore do not give rise to gaugino masses (or μ, $B\mu$, and A terms). The simplest way to generate gaugino masses is to assume the standard-model gauge fields propagate in the bulk. Gaugino masses can then be generated by contact terms of the form eq. (1.2) with the hidden sector brane. More precisely, the term is the supersymmetric completion of the operator

$$\Delta\mathcal{L}_D = \delta^{D-4}(y-y_0)\Sigma(x)\,\mathrm{tr}(F^{MN}F_{MN})(x,y_0) + \cdots\,, \tag{1.7}$$

where Σ is the scalar component of a chiral superfield propagating on the 3-brane at $y = y_0$, F^{MN} is the field strength of the standard-model gauge field.

There are several possibilities for generating μ and $B\mu$ terms. One possibility is to assume that the Higgs fields propagate in the bulk. Then μ and $B\mu$ terms can be generated by contact terms with the hidden sector of the form eq. (1.3); more precisely, the supersymmetric completion of the operators

$$\Delta\mathcal{L}_D = \delta^{D-4}(y-y_0)\left[(\partial^2\Sigma^\dagger)(x) + (\partial^2\Sigma^\dagger\Sigma)(x)\right](H_u H_d)(x,y_0) + \text{h.c.} + \cdots\,. \tag{1.8}$$

This leads to phenomenology similar to conventional hidden sector models, except that A terms can be naturally be small because separating the hidden and visible sectors forbids operators of the form eq. (1.4). (There are small loop-suppressed A terms from anomaly-mediation [7].)

Another possibility is that there is a singlet field S whose VEV generates effective μ and $B\mu$ terms [12]. We assume that the 4-dimensional effective theory includes the cubic superpotential terms

$$\Delta W = \lambda S H_u H_d + \frac{\kappa}{3}S^3\,. \tag{1.9}$$

The S^3 term is not $\mathrm{U}(1)_X$ invariant, but non-invariant terms such as this can be present below the scale where $\mathrm{U}(1)_X$ is broken. (We will see that this requires the Higgs fields to propagate in the bulk, while S can propagate either in the bulk or on the hidden-sector brane.) The fields $H_{u,d}$ and S can naturally have a negative mass-squared of order the weak scale, giving rise to realistic electroweak symmetry breaking. An attractive feature of this model in the present higher-dimensional context is that it automatically solves the SUSY CP problem [13]: all CP-violating phases can be rotated into the CKM phase and ϑ_{QCD}. For this it is crucial that there are no uncontrolled A terms from higher-dimension operators.

The couplings of the bulk fields such as the gauge fields (and possibly Higgs fields) in the effective 4-dimensional theory will be suppressed by the volume of the compact subspace, so r cannot be too large. On the other hand, we have seen above that r must be sufficiently large so that FCNC's are suppressed. Since the suppression of FCNC's is exponential, these requirements are easily met, especially for a small number of large extra dimensions. We will show below that even for a large number of extra dimensions (e.g. $D = 11$) these requirements can be met if the standard-model gauge couplings are strongly coupled at the fundamental Planck scale.

In order that the visible sector scalar masses be close to the gaugino masses, the VEV v that breaks $\mathrm{U}(1)_X$ must also be near the fundamental Planck scale. This emerges naturally if all couplings are order 1 in units of the fundamental Planck scale. (In fact, the bulk standard-model gauge couplings must be somewhat larger than this in order for the effective 4-dimensional gauge couplings to be order 1, but this factor need not be larger than an order of magnitude.)

Alternatively, we can consider a scenario where *all* microscopic couplings are near their strong-coupling values at the fundamental scale. In this scenario, the only large parameter is the size of the compact dimensions, which need only be an order of magnitude larger than the fundamental scale. We carry out estimates of the size of parameters in this scenario, paying attention to geometrical factors that control the size of loop graphs ('naïve dimensional analysis' [17, 18]). The result is that this scenario naturally gives scalar masses, gaugino masses, and μ and $B\mu$ terms of realistic size without the introduction of small parameters.

This strongly-coupled scenario is particularly appealing in the light of the modern view of string theory as a single connected moduli space of different theories, with the known 10-dimensional superstring theories and 11-dimensional SUGRA appearing as weak coupling limits [14]. Already in the early days of string theory it was realized that it is extremely difficult to find phenomenologically viable vacua near weak coupling because the theory generally runs away to zero coupling [15]. With the modern picture in mind, one can conjecture that phenomenologically viable vacua exist in the regime where the theory has no weak-coupling description. But then the absence of FCNC's appears especially puzzling, since we expect all operators allowed by gauge symmetries to be generated with approximately equal strength. The present class of models gives a possible solution: if the vacuum contains 3-branes, and some compact dimensions are an order of magnitude larger than the string scale, this can act as a 'seed' for accidental symmetries in the low-energy world. There are other small parameters that are not directly explained in this approach (such as the small Yukawa couplings); it would be interesting to see whether these small parameters can also have a geometric origin in a scenario of this type.

This paper is organized as follows. In section 2, we discuss the physics of breaking gauge symmetry on branes. The considerations are elementary, but there are some surprises. In section 3, we write down explicit models and give estimates for soft masses. section 4 contains our conclusions.

2. Breaking gauge symmetry on branes

In this section, we discuss the breaking of a bulk gauge symmetry by the VEV's of charged fields propagating on 3-branes. This is simpler than breaking the gauge symmetry in the bulk because the allowed interactions of supersymmetric theories in higher dimensions are quite limited.

We will be interested in operators in the 4-dimensional low-energy theory of the form

$$\Delta\mathcal{L}_4 \sim \int d^4\theta\, \Sigma^\dagger\Sigma \left[Q^\dagger Q + H^\dagger H\right], \tag{2.1}$$

where Σ is a field propagating on the hidden sector brane, Q is a field propagating on the visible sector brane, and H is a bulk field. To compute the coefficient of quartic terms such as these, it suffices to compute the 4-fermion component. Below we will compute the contribution to the 4-fermion term from the tree-level exchange of vector bosons, where the couplings are completely determined by gauge invariance. In supersymmetric theories in more than 4 dimensions, there are additional propagating bosonic fields in the gauge multiplet that could in principle contribute to the coefficient of the 4-fermion term, but we will later give an explicit example where we show that only the vector boson contributes. We believe that this feature is more general, but establishing this would require a general understanding of the couplings between higher-dimensional supersymmetric gauge fields to branes. We will not address this question here.

We consider two parallel 3-branes in a D-dimensional space, with $D-4$ dimensions compactified on a length scale r. We assume that there is a $\mathrm{U}(1)_X$ gauge field in the bulk, and charged fermions on the branes and in the bulk.[3] The lagrangian is

$$\begin{aligned}\mathcal{L}_D = &-\frac{1}{4g_D^2}(F^{MN}F_{MN})(x,y) + \bar{\Psi}_B i\Gamma^M D_M\Psi_B + \\ &+\delta^{D-4}(y-y_1)\left[(D^\mu\phi^\dagger D_\mu\phi)(x) - V(\phi(x)) + (\bar{\psi}_1 i\,\gamma^\mu D_\mu\psi_1)(x) + \cdots\right] + \\ &+\delta^{D-4}(y-y_2)\left[(\bar{\psi}_2 i\,\gamma^\mu D_\mu\psi_2)(x) + \cdots\right].\end{aligned} \tag{2.2}$$

[3] The conclusions of this section hold for non-abelian gauge theories as well.

Here $M, N, \ldots = 0, \ldots, D-1$ are Lorentz indices for the D-dimensional spacetime, x^μ ($\mu = 0, \ldots, 3$) are coordinates along the 3-brane, and y^I ($I = 4, \ldots, D-1$) are coordinates for the compact space. We will assume that $\langle \phi \rangle \neq 0$, and we will work out the interactions between the fermions induced by $\mathrm{U}(1)_X$ gauge exchange. For the bulk fermion, we are interested only in the zero mode

$$\Psi_B(x, y) = \frac{1}{\sqrt{V_{D-4}}} \psi_B(x) + \cdots, \tag{2.3}$$

where V_D is the volume of the compact space. The gauge fields A_M are normalized to have mass dimension $+1$, so that covariant derivatives are given by

$$D_M = \partial_M - i A_M X, \tag{2.4}$$

where X is the $\mathrm{U}(1)_X$ charge matrix. The gauge coupling g_D has mass dimension $(4-D)/2$.

The lagrangian (2.2) makes sense as an effective lagrangian valid below an ultraviolet cutoff Λ_0 provided that we include all counterterms allowed by the symmetries proportional to powers of Λ_0. In particular, the lagrangian will include terms of the form

$$\Delta \mathcal{L}_D \sim \frac{(\bar{\Psi}_B \Psi_B)^2}{\Lambda_0^{D-2}} + \delta^{D-4}(y - y_1) \left[\frac{(\bar{\psi}_1 \psi_1)^2}{\Lambda_0^2} + \frac{(\bar{\psi}_1 \psi_1)(\bar{\Psi}_B \Psi_B)}{\Lambda_0^{D-2}} + \frac{(\bar{\Psi}_B \Psi_B)^2}{\Lambda_0^{2(D-3)}} \right] + (1 \leftrightarrow 2), \tag{2.5}$$

so that contact terms of this form are not calculable unless their coefficients are parametrically larger than those above. Note that the D-dimensional theory cannot have contact term of the form $(\bar{\psi}_1 \psi_1)(\bar{\psi}_2 \psi_2)$ by locality, so interactions of this form are always calculable.

We assume that the scalar field on the brane at $y = y_1$ gets a VEV that breaks the gauge symmetry:

$$\langle \phi \rangle = \frac{v}{\sqrt{2}}. \tag{2.6}$$

It will be convenient for us to normalize the $\mathrm{U}(1)_X$ charges by taking the scalar field ϕ (and hence v) to have charge $+1$.

To fix the gauge, we add a bulk gauge-fixing term proportional to $(\partial^M A_M)^2$ to make the gauge kinetic term canonical:

$$\mathcal{L}_D = -\frac{1}{2 g_D^2} \partial^M A^N \partial_M A_N + \cdots. \tag{2.7}$$

This does not completely fix the gauge, since we can still make gauge transformations with gauge parameter α satisfying $\partial^M \partial_M \alpha = 0$. We can use this residual gauge freedom to choose unitary gauge for the scalar field on the brane at $y = y_1$. In this gauge, we can expand the gauge fields in Kaluza-Klein (KK) modes:

$$A^\mu(x, y) = \sum_k A_k^\mu(x) f_k(y), \qquad A^I(x, y) = \sum_k A_k^I(x) g_k(y), \tag{2.8}$$

where the KK wavefunctions satisfy

$$\begin{aligned} -\nabla_\perp^2 f_k(y) + g_D^2 v^2 \delta^{D-4}(y - y_1) f_k(y) &= m_k^2 f_k(y)\,, \\ -\nabla_\perp^2 g_k(y) &= \tilde{m}_k^2 g_k(y)\,, \end{aligned} \tag{2.9}$$

where $\nabla_\perp^2$ is the laplacian in the compact space, and m_k, $\tilde{m}_k$ are the masses of the KK modes.

Note that the VEV on the 3-brane does not affect the A^I fields, so there are in general $D-4$ massless scalars in the 4-dimensional theory. In a supersymmetric theory, there are additional fields in the bulk gauge multiplet (e.g. gauginos) that do not acquire a mass from the VEV on the 3-brane. This can be avoided by an orbifold projection that gives mass to these fields, as in the specific model discussed in section 3.1. More generally, some of these states may survive to the weak scale and have phenomenological consequences. This will be discussed in section 3.4.

The effect of the VEV on the fields A^μ depends on the spacetime dimension D as well as the choice of the parameters. The dimensionless measure of the relevance of the delta function term as a perturbation is

$$\epsilon \equiv \frac{g_D^2 v^2 r^2}{V_{D-4}} \sim g_4^2 v^2 r^2\,. \tag{2.10}$$

We will always be interested in $r \gg 1/v$ ('large' extra dimensions), so we see that $\epsilon \gg 1$ if g_D is large enough so that $g_4 \sim 1$. (We will later show that this is possible even for large D.) However $\epsilon \ll 1$ if $g_D^2 \sim 1/v^{D-4}$ for $D \geq 6$. It is therefore natural to consider both large and small ϵ for arbitrary dimensions.

If $\epsilon \ll 1$, we can use perturbation theory to find the mass of the lightest KK mode. The unperturbed KK wavefunction is simply a constant zero mode, and we find

$$m_0^2 = \frac{g_D^2 v^2}{V_{D-4}} = g_4^2 v^2\,, \tag{2.11}$$

where g_4 is the gauge coupling in the 4-dimensional effective theory. This is the result we would expect from the Higgs mechanism in the 4-dimensional low-energy theory. Intuitively, the picture is that the zero mode of the gauge field is constant across the extra dimension, and therefore feels the VEV on the other wall as if it were 4-dimensional.

For $\epsilon \gg 1$, we cannot treat the delta function as a perturbation and the situation is quite different. We can understand this limit intuitively by noting that the 3-brane on which the $\mathrm{U}(1)_X$ is broken is a superconductor. We can compute the mass of the lightest KK mode from the fact that it controls the exponential fall-off of the electric field due to a point charge at distances large compared to r. We expect a non-zero cutoff-independent mass, since the result is already nonzero and cutoff-independent for $\epsilon \ll 1$. This is illustrated in figure 1 for the case $D = 5$. The field will be

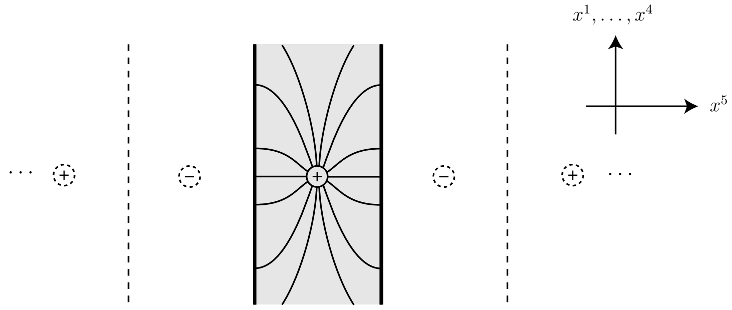

Figure 1: Electric field due to a point charge in a space with one compact dimension. The two solid vertical lines represent two copies of the 3-brane where $\mathrm{U}(1)_X$ is broken; the physical space is the shaded region between them. The field can be reproduced by an infinite number of image charges of alternating signs, as shown.

distorted by induced surface charges on the superconducting 3-brane that tend to screen the electric field. In the limit $\epsilon \to \infty$ the brane acts as a perfect conductor, and the problem of finding the electric field of a point charge is purely geometric, with the superconducting 3-brane acting as a boundary condition. Just by dimensional analysis the only possible result is

$$m_0^2 \sim \frac{1}{r^2}\,. \tag{2.12}$$

In fact, for $D = 5$ the electrostatics problem described here can be solved by the method of images (see figure 1). The infinite number of image charges is responsible for the exponential fall-off of the electric field. For finite ϵ, we expect corrections to this picture suppressed by inverse powers of ϵ.

The qualitative considerations above are born out by direct calculation of the KK decomposition. For example, for $D = 5$ with the extra dimension compactified on a circle of radius r, we have

$$f_k(y) = \sin\left(\frac{ky}{r}\right) + \frac{2k}{g_5^2 v^2 r}\cos\left(\frac{ky}{r}\right), \qquad y \le 0 < 2\pi r\,, \tag{2.13}$$

where

$$m_k^2 = \frac{k^2}{r^2} \tag{2.14}$$

and the allowed values of k are determined by

$$k\tan(\pi k) = \frac{1}{2} g_5^2 v^2 r\,. \tag{2.15}$$

For $\epsilon = g_5^2 v^2 r \gg 1$, the solutions are

$$k = \left(n + \frac{1}{2}\right)\left[1 + O(\epsilon^{-1})\right], \qquad n = 0, 1, \ldots \tag{2.16}$$

so the masses of the lowest-lying KK modes are of order $1/r$ as claimed.

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For large values of D the KK decomposition is more complicated, and it is simpler to compute the contact terms we are interested in directly from the higher-dimensional theory. We perform the tree-level calculation by solving the equations of motion in the compact directions

$$-\frac{1}{g_D^2}\nabla_\perp^2 A^\mu + \delta^{D-4}(y-y_1)\left[v^2 A^\mu + J_1^\mu\right] + \delta^{D-4}(y-y_2)J_2^\mu + \frac{J_B^\mu}{V_{D-4}} = 0\,, \qquad (2.17)$$

where

$$J_{1,2}^\mu = \bar\psi_{1,2}\gamma^\mu X\psi_{1,2}\,, \qquad J_B^\mu = \bar\psi_B\gamma^\mu X\psi_B\,, \qquad (2.18)$$

are treated as background fields. We are mainly interested in the coefficient of the $J_1 J_2$ term, and we claim that for any dimension and for any value of ϵ, this coefficient is of order $1/v^2$. If $\epsilon \ll 1$, this is not surprising since the lightest KK mode has mass of order $g_4^2 v^2$, and exchange of this mode gives rise to a contact interaction with strength $g_4^2/(g_4^2 v^2) \sim 1/v^2$. For $\epsilon \gg 1$ this result is perhaps counterintuitive since one might expect the coefficient to be $g_4^2/m_{\rm KK}^2 \sim g_D^2/r^{D-6}$.

We begin with $D = 5$ compactified on a circle of radius r, with the branes at $y = 0$ and $y = \pi r$. The solution is

$$A^\mu(y) = \begin{cases} \alpha_1^\mu + \beta_1^\mu y + \dfrac{1}{4\pi r} g_5^2 J_B^\mu y^2\,, & 0 \le y < \pi r\,, \\ \alpha_2^\mu + \beta_2^\mu y + \dfrac{1}{4\pi r} g_5^2 J_B^\mu y^2 & \pi r \le y < 2\pi r\,, \end{cases} \qquad (2.19)$$

where the coefficients $\alpha_{1,2}$ and $\beta_{1,2}$ are determined by matching the value of A^μ and the discontinuity in its derivative at the positions of the delta functions. The result of solving the equations and substituting back into the lagrangian is

$$\Delta\mathcal{L}_{4,\rm eff} = -\frac{1}{2v^2}(J_1 + J_2 + J_B)^2 - \frac{\pi g_5^2 r}{4}\left(J_2^2 + J_2 J_B + \frac{1}{3}J_B^2\right). \qquad (2.20)$$

Note that the coefficient of the $J_1 J_2$ and $J_1 J_B$ terms is of order $1/v^2$ rather than $g_4^2/m_{\rm KK}^2 \sim g_D^2/r^{D-6} \gg 1/v^2$, as might be expected. This is because the overlap of the low-lying KK modes with the brane where $\mathrm{U}(1)_X$ is broken is suppressed. eq. (2.13) shows that the this overlap is suppressed by $1/\epsilon = 1/(g_5^2 v^2 r)$ and the contribution of a low-lying KK mode to the $J_1 J_2$ or $J_1 J_B$ term is of order

$$\frac{g_4^2/\epsilon}{m_{\rm KK}^2} \sim \frac{1}{v^2}\,. \qquad (2.21)$$

We next consider $D = 6$ compactified on a sphere with the branes at opposite poles. The classical solution for A^μ then depends only on the azimuthal angle θ. To make the equations well-defined, the delta functions must be regulated. We do this by replacing the point-like delta functions by a delta function ring of finite radius. Specifically, in terms of the variable $z \equiv \cos\theta$ we have

$$\delta^2(y - y_{1,2}) \to \frac{1}{2\pi r^2}\delta(z - z_\pm)\,, \qquad z_\pm \equiv \pm(1-\delta)\,, \qquad (2.22)$$

where $\delta > 0$ is a small cutoff parameter. The equations of motion are

$$-\frac{1}{g_6^2}\frac{d}{dz}\left[(1-z^2)\frac{dA^\mu}{dz}\right] + \frac{1}{2\pi}\delta(z-z_+)\left[v^2 A^\mu + J_1^\mu\right] + +\frac{1}{2\pi}\delta(z-z_-)J_2^\mu + \frac{1}{4\pi}J_B^\mu = 0\,. \tag{2.23}$$

Note that r factors out; this system is classically scale invariant. The solution is

$$A^\mu(z) = \begin{cases} \alpha_1^\mu - \dfrac{g_6^2}{4\pi}J_B^\mu \ln(1-z) & -1 \le z < z_-, \\ \alpha^\mu + \beta^\mu \ln\dfrac{1+z}{1-z} - \dfrac{g_6^2}{8\pi}J_B^\mu \ln(1-z^2) & z_- \le z \le z_+, \\ \alpha_2^\mu - \dfrac{g_6^2}{4\pi}J_B^\mu \ln(1+z) & z_+ < z \le 1. \end{cases} \tag{2.24}$$

As before, the coefficients $\alpha_{1,2}$ and α, β are determined by matching the value of A^μ and the discontinuity in its derivative at the delta functions. Substituting back into the lagrangian, we obtain

$$\begin{aligned}\Delta\mathcal{L}_{\text{eff},4} = &-\frac{1}{2v^2}(J_1+J_2+J_B)^2 + \left(\frac{g_6^2}{4\pi}\ln\frac{\delta}{2}\right)(J_2^2 + J_2 J_B) + \\ &+\frac{g_6^2}{8\pi}\left(1+\ln\frac{\delta}{2}\right)J_B^2 + O(\delta).\end{aligned} \tag{2.25}$$

The physical origin of the logarithmic divergences is that the brane acts as a source for modes with wavelength of order the brane thickness. The logarithmic divergences can be absorbed into counterterms of the form eq. (2.5). Note that, as in the $D = 5$ case, all contact terms involving J_1 are of order $1/v^2$ for arbitrary ϵ. Although the spherical geometry is clearly a special case, we believe that the qualitative features are quite general.[4]

It is easy to generalize this calculation to higher dimensions in the case where the compact space is a $(D-4)$-sphere of radius r and the positions of the 3-branes are at opposite poles. We do not include a charged bulk fermion for simplicity. As before we use the variable $z = \cos\theta$ where θ is the azimuthal angle, and regulate the delta functions by replacing them with delta function rings at $z_\pm = \pm(1-\delta)$. The equations of motion are

$$\begin{aligned}&-\frac{1}{(1-z^2)^{(D-6)/2}}\frac{\partial}{\partial z}\left[(1-z^2)^{(D-4)/2}\frac{\partial A_\mu}{\partial z}\right] + \\ &+\frac{g_D^2}{r^{D-6}}\frac{\delta(z-z_+)}{\Omega_{D-4}(1-z_+^2)^{(D-6)/2}}\left[v^2 A_\mu + J_{1\mu}\right] + \\ &\qquad +\frac{g_D^2}{r^{D-6}}\frac{\delta(z-z_-)}{\Omega_{D-4}(1-z_-^2)^{(D-6)/2}}J_{2\mu} = 0\,,\end{aligned} \tag{2.26}$$

[4]To be realistic, a spherical geometry would require a source for the curvature of spacetime. In a supersymmetric theory, such a source of curvature would in general break SUSY. We do not enter into these considerations, since we are using the spherical geometry only for illustrative purposes.

where $\Omega_n = 2\pi^{n/2}/\Gamma(n/2)$ is the volume of the unit $(n-1)$-sphere. The solution is

$$A_\mu(y) = \begin{cases} \alpha_{1\mu} & -1 \le z < z_- \,, \\ \alpha_\mu + \beta_\mu f_D(z) & z_- \le z \le z_+ \,, \\ \alpha_{2\mu} & z_+ < z \le 1 \,, \end{cases} \tag{2.27}$$

where

$$f_D(z) = \int^z dz \, \frac{1}{(1-z^2)^{(D-4)/2}} \,. \tag{2.28}$$

The coefficients $\alpha_{1,2}$ and α, β can be determined by matching the value of A^μ and the discontinuity in its derivative at the delta function shells. Substituting back into the lagrangian, we obtain

$$\Delta\mathcal{L}_{\text{eff},4} = -\frac{1}{2v^2}(J_1 + J_2)^2 - \frac{g_D^2 f_D(z_-)}{2\Omega_{D-4} r^{(D-6)/2}} J_2^2 \,. \tag{2.29}$$

The coefficient of $J_2 J_2$ diverges in the limit $\delta \to 0$. In terms of a physical length cutoff (brane thickness) a, we have $\delta \sim (a/r)^{1/2}$ and hence $f_D(z_-) \sim (r/a)^{(D-6)/2}$. The coefficient of the J_2^2 term is therefore of order $g_D^2/a^{(D-6)/2}$. This divergence can be absorbed by a counterterm of the form eq. (2.5).

We have been considering contact terms between fields on the brane where $\mathrm{U}(1)_X$ is broken and a spatially separated brane. However, we could consider contact terms between fields on spatially separated branes, neither of which is the one on which the $\mathrm{U}(1)_X$ is broken. In that case, there is no suppression of the KK wavefunctions and the coefficient of $J_2 J_3$ is of order g_4^2/m_{KK}^2, as expected.

3. Realistic models

We now apply the results above to the construction of realistic models.

3.1 An explicit model

We begin by giving an explicit model in 5 spacetime dimensions as an existence proof. We believe that these ideas can be made to work in more general settings (e.g. in models with additional 'large' dimensions).

We assume that the 5-dimensional theory has minimal supersymmetry, namely 8 real supercharges. One dimension is compactified on a Z_2 orbifold with the hidden and visible sector fields localized on the orbifold fixed points. The orbifold projection explicitly breaks half the supersymmetry, which gives unbroken $\mathcal{N} = 1$ SUSY in the 4-dimensional effective theory. This makes it simple to construct couplings to the orbifold boundary, since these need only preserve the unbroken $\mathcal{N} = 1$ supersymmetry. This set-up was analyzed in ref. [19], and we make use of the formalism described in that paper. This scenario is also closely related to the one advocated by Hořava and Witten in the context of M theory [6].

There are two types of 5-dimensional multiplets used to construct the model, and we now describe them briefly. We use the conventions of ref. [19], which should be consulted for more detail. A gauge multiplet $(\Phi, A_M, \lambda^j, X^a)$ consists of a real scalar Φ, a gauge field A_M, a symplectic Majorana spinor λ^j ($j = 1, 2$), and real auxiliary fields X^a ($a = 1, 2, 3$). The indices j and a are doublet and triplet indices for an $\mathrm{SU}(2)_R$ symmetry. The fields that are even under the orbifold parity form an $\mathcal{N} = 1$ gauge multiplet (A_μ, λ^1_L, D), where

$$D \equiv X^3 - \partial_5 \Phi \tag{3.1}$$

is the auxiliary field. (Note that this formalism forces us to use Wess-Zumino gauge for the induced $\mathcal{N} = 1$ gauge multiplet.)

A 5-dimensional hypermultiplet (ϕ^j, ψ, Y^j) consists of 2 complex scalars ϕ^j, a Dirac spinor ψ, and 2 complex auxiliary fields Y^j. The fields that are even under the orbifold parity form an $\mathcal{N} = 1$ chiral multiplet (ϕ^1, ψ_L, F), where

$$F \equiv Y^1 - \partial_5 \phi^2 \tag{3.2}$$

is the auxiliary field.

These results make it simple to couple even-parity bulk fields to 4-dimensional fields propagating on the boundary. For example, the coupling of the bulk $\mathrm{U}(1)_X$ gauge field to charged fields on the boundary can be written

$$\begin{aligned} \Delta\mathcal{L}_5 &= \delta(y - y_1) \int d^4\theta\, Q^\dagger e^{VX} Q \\ &= \delta(y - y_1) \left[(D^\mu \tilde{Q}^\dagger D_\mu \tilde{Q}) + Q^\dagger i\sigma^\mu D_\mu Q + (X^3 - \partial_5\Phi) \tilde{Q}^\dagger X \tilde{Q} \right]. \end{aligned} \tag{3.3}$$

where V denotes the even-parity $\mathcal{N} = 1$ gauge multiplet obtained from the bulk $\mathrm{U}(1)_X$ multiplet, evaluated at $y = y_1$.

The couplings above are to be used to compute the contributions from exchange of massive $\mathrm{U}(1)_X$ gauge fields between the orbifold boundaries. From eq. (3.3) we see that only A_μ couples to the fermions, so the calculations of section 2 give the correct coefficient of the resulting 4-fermion terms.

Note that the vector polarization A_5 and the 'extra' gaugino λ^2 have masses of order $1/r$ by the orbifold projection. The $\mathcal{N} = 1$ gaugino λ^1 gets a mass from the supersymmetric Higgs mechanism (since SUSY is not broken by the boundary VEV). Therefore, there are no extra light states in this model.

We now consider the higher-dimension operators that couple the bulk and boundary fields. We will consider the case where the Higgs fields propagate in the bulk in order to illustrate the required couplings. We assume that each Higgs multiplet arises from a separate bulk hypermultiplet. We are interested in the higher-dimension couplings

$$\begin{aligned} \Delta\mathcal{L}_5 \sim \delta(y - y_2) \bigg\{ &\int d^4\theta \left[\Sigma^\dagger H_u H_d + \Sigma^\dagger \Sigma (H_u H_d + H_u^\dagger H_u + H_d^\dagger H_d) \right] + \\ &+ \int d^2\theta\, \Sigma W^\alpha W_\alpha + \text{h.c.} \bigg\}, \end{aligned} \tag{3.4}$$

where $H_{u,d}$ are $\mathcal{N} = 1$ chiral multiplets arising from the even-parity components of the bulk Higgs fields, and W_α is the $\mathcal{N} = 1$ standard-model gauge field strength arising from the even-parity components of the bulk gauge multiplet. Upon matching to the effective 4-dimensional theory, these operators give rise to effective operators of the form eqs. (1.2) and (1.3). These operators give rise to gaugino masses and μ and $B\mu$ terms when SUSY is broken by $F_\Sigma \neq 0$.

We now turn to squark and slepton masses. If $\mathrm{U}(1)_X$ is broken in the visible sector, there are flavor-violating operators of the form

$$\int d^4\theta \, \frac{c_{jk}}{M^{2n}} (\Phi^\dagger e^{VX_\Phi} \Phi)^n Q_j^\dagger e^{VX_Q} Q_k \,, \tag{3.5}$$

where Φ is the field whose VEV breaks $\mathrm{U}(1)_X$. Since (as we will see) $\langle \Phi \rangle = v \sim M$, these operators are unsuppressed at low energies. There is no reason for these operators to conserve flavor, so these operators will give rise to generation-dependent couplings of the $\mathrm{U}(1)_X$ boson.[5] We must therefore break $\mathrm{U}(1)_X$ on the hidden sector brane. In that case, squark and slepton masses are generated by the operator

$$\Delta \mathcal{L}_4 \sim \frac{1}{v^2} \int d^4\theta \, (\Sigma^\dagger X \Sigma)(Q^\dagger X Q) \tag{3.6}$$

generated by $\mathrm{U}(1)_X$ exchange. This term will conserve flavor if $\mathrm{U}(1)_X$ commutes with the flavor symmetry.

Since we want all the squark and slepton masses to be positive, the signs of the $\mathrm{U}(1)_X$ charges of all squark and sleptons must be the same. This means that the $\mathrm{U}(1)_X$ gauge field necessarily has mixed anomalies with the standard-model gauge group. This is not inconsistent because the $\mathrm{U}(1)_X$ gauge group is broken at the scale $v \sim M$, so the consistency of the low-energy field theory is really all that is required if one is willing to put off the derivation of the model from string theory. However, it is reassuring to note that there is no difficulty in constructing field theories above the scale v that are free from gauge anomalies. Anomalies of the type $\mathrm{U}(1)_X\, \mathrm{SU}(3)_C^2$, $\mathrm{U}(1)_X\, \mathrm{SU}(2)_W^2$, and $\mathrm{U}(1)_X\, \mathrm{U}(1)_Y^2$ can be canceled by adding chiral fields that are in vector-like representations of the standard-model gauge group, but chiral with respect to $\mathrm{U}(1)_X$. Anomalies of the type $\mathrm{U}(1)_X^2\, \mathrm{U}(1)_Y$ can be simply canceled if all fields charged under the standard-model have the same value of the $\mathrm{U}(1)_X$ charge, in which case the cancellation of these anomalies follows from the relation $\mathrm{tr}(Y) = 0$. All the extra fields added in this way can obtain $\mathrm{U}(1)_X$-violating masses at the scale v.[6]

We now consider the $\mathrm{U}(1)_X$ invariance of the visible sector Yukawa terms. Because $\mathrm{U}(1)_X$ is broken in the hidden sector, the simplest possibility is that the Yukawa

[5] We thank R. Rattazzi for pointing this out to us.

[6] Since $\mathrm{U}(1)_X$ and the standard-model gauge groups propagate in the bulk, the fields that cancel the anomalies can be localized on a distance wall.

terms are invariant under $\mathrm{U}(1)_X$. For the small Yukawa couplings one can contemplate the possibility that the Yukawa couplings arise from small $\mathrm{U}(1)_X$ breaking effects (such as the VEV of a 'flavon' field charged under $\mathrm{U}(1)_X$). However, it seems implausible that the order-1 top Yukawa coupling arises in this way. If we assume that the top Yukawa coupling is $\mathrm{U}(1)_X$ invariant, then there is a contribution to the up-type Higgs scalar mass-squared

$$\Delta m_{Hu}^2 = -(m_{\tilde{t}L}^2 + m_{\tilde{t}R}^2)\,. \tag{3.7}$$

If the other Yukawa couplings also arise from $\mathrm{U}(1)_X$ invariant effects we have

$$\Delta m_{Hu}^2 = -(m_{\tilde{Q}L}^2 + m_{\tilde{u}R}^2)\,, \qquad \Delta m_{Hd}^2 = -(m_{\tilde{Q}L}^2 + m_{\tilde{d}R}^2)\,. \tag{3.8}$$

(The squark and slepton masses of different generations are universal.) Given experimental bounds on squark masses, this may require moderate fine-tuning of other contributions to the Higgs masses (e.g. from the μ term) to obtain realistic electroweak symmetry breaking.

Our next task is to estimate the size of soft SUSY breaking in this model. To do this we will need to know how to estimate parameters in strongly-coupled theories in higher dimensions, and we address this question next.

3.2 Naïve dimensional analysis in higher dimensions

We argued in the Introduction that it is an attractive possibility that all the couplings of the theory are strong at a fundamental scale Λ, which may be identified with the fundamental scale in strongly-coupled M theory. Apart from this, it is important to estimate the maximum possible value of the D-dimensional gauge coupling, since this determines the maximum size of the extra dimensions. With this motivation, we explain how to estimate the size of terms in the effective theory under the assumption that the fundamental theory is strongly coupled and contains no small parameters, generalizing previous results [17, 18] to theories in higher dimensions with branes. In such theories, one might expect that all couplings in the effective theory below the scale Λ are of order 1 in units of Λ. However, experience with QCD and exactly solvable supersymmetric models [16] shows that there are large hierarchies in the effective couplings when they are expressed in units of the scale where the theory is strongly coupled. As we will explain, these can be understood from the condition that all interactions in the effective theory get strong at the same scale. These factors are related to the phase-space factors in loop integrals, and are therefore strongly dimension-dependent, so the generalization is non-trivial.

In a D-dimensional theory, a typical loop integral can be written

$$\int \frac{d^D P}{(2\pi)^D}\, f(P^2) \sim \frac{\Omega_D}{2(2\pi)^D} \int dP^2\, P^{D-2} f(P^2)\,. \tag{3.9}$$

This means that every loop factor is kinematically suppressed by a factor of order

$$\frac{1}{\ell_D} = \frac{1}{2^D \pi^{D/2} \Gamma(D/2)} \,. \tag{3.10}$$

We now estimate the size of couplings in the D-dimensional effective theory assuming that at energies $E \sim \Lambda$, loops of all kinds are unsuppressed. The effective theory is perturbative for $E \ll \Lambda$ because of kinematic suppressions. We can immediately write down the form of the lagrangian under this assumption by noting that an overall factor in front of the lagrangian acts as a loop-counting parameter, like $\hbar$ in the semiclassical expansion. Therefore, the lagrangian takes the form

$$\mathcal{L}_D \sim \frac{\Lambda^D}{\ell_D} \hat{\mathcal{L}}_{\rm bulk} \left(\hat{\Phi}, \frac{\partial}{\Lambda} \right) + \delta^{D-4}(y) \frac{\Lambda^4}{16\pi^2} \hat{\mathcal{L}}_{\rm brane} \left(\hat{\phi}, \hat{\Phi}, \frac{\partial}{\Lambda} \right) . \tag{3.11}$$

Here, $\hat{\Phi}$ is a bulk field, $\hat{\phi}$ is a brane field, and all couplings in the 'reduced lagrangians' $\hat{\mathcal{L}}_{\rm bulk}$ and $\hat{\mathcal{L}}_{\rm brane}$ are order 1.

For example, comparing the form of the effective lagrangian eq. (3.11) to the definition of the D-dimensional Planck scale

$$\mathcal{L}_D = -\frac{1}{2} M_D^{D-2} \nabla^{(D)} + \cdots \tag{3.12}$$

immediately gives

$$\Lambda \sim \ell^{1/(D-2)} M_D \,. \tag{3.13}$$

Numerically, $\Lambda \sim 10 M_D$ for $5 \le D \le 11$.

The fields $\hat{\Phi}$ and $\hat{\phi}$ appearing in eq. (3.11) have been taken to be dimensionless; their kinetic terms have the form

$$\mathcal{L}_D \sim \frac{\Lambda^{D-2}}{\ell_D} (\partial \hat{\Phi})^2 + \delta^{D-4}(y) \frac{\Lambda^2}{16\pi^2} (\partial \hat{\phi})^2 + \cdots \,. \tag{3.14}$$

Fields with canonical kinetic terms can be defined by

$$\Phi \sim \frac{\Lambda^{(D-2)/2}}{\sqrt{\ell_D}} \hat{\Phi}, \qquad \phi \sim \frac{\Lambda}{4\pi} \hat{\phi}. \tag{3.15}$$

When the lagrangian eq. (3.11) is expressed in terms of canonical fields, the interactions contain nontrivial geometrical factors.

The prefactors in the lagrangian eq. (3.11) give rise to enhancement factors for loops that cancel the kinematic loop suppression factors. This is clear for diagrams involving only bulk fields or only brane fields. For diagrams with bulk fields coupled to brane fields, this is less obvious, and we will discuss this point briefly. The simplest set-up where we can understand eq. (3.11) is where all D dimensions are non-compact. Since the coefficients we are estimating arise at the scale $\Lambda \gg 1/r$, this approximation is sufficient. The theory may contain linear or quadratic terms

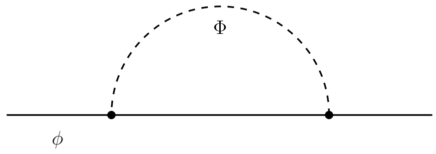

Figure 2: One-loop diagram involving both bulk fields Φ and brane fields ϕ.

in the bulk fields localized on the wall, such as $\Delta\hat{\mathcal{L}}_{\text{brane}} \sim \hat{\Phi} + \hat{\Phi}\hat{\phi} + \hat{\Phi}^2$. We will treat these as perturbations for purposes of estimating the kinematic factors. In this case, the propagators for the bulk and brane fields are exactly the same as in theories without branes:

$$\begin{aligned}
\langle \Phi(X_1)\Phi(X_2)\rangle &= \int \frac{d^D P}{(2\pi)^D} \frac{i}{P^2 - M^2} e^{-iP\cdot(X_1 - X_2)}, \\
\langle \phi(x_1)\phi(x_2)\rangle &= \int \frac{d^4 k}{(2\pi)^4} \frac{i}{k^2 - m^2} e^{-ik\cdot(x_1 - x_2)}.
\end{aligned} \tag{3.16}$$

(Naïve dimensional analysis suggests that M and m are either vanishing or of order Λ.) The Feynman rules for the momentum-space correlation functions are also the same as in theories without branes, except that in couplings of bulk fields to the brane, the momenta perpendicular to the brane is not conserved. If the bulk field line coming from the brane is part of a loop, the perpendicular components of the momenta are freely integrated over.

For example, consider a coupling (written in terms of canonically normalized fields)

$$\Delta\mathcal{L}_D \sim \delta^{D-4}(y) \frac{\sqrt{\ell_D}}{\Lambda^{(D-6)/2}} \phi^2 \Phi\,. \tag{3.17}$$

We can then consider a 'mixed' loop diagram such as the one shown in figure 2. Using the rules above, we obtain

$$\begin{aligned}
\text{figure 2} &= \left(\frac{\sqrt{\ell_D}}{\Lambda^{(D-6)/2}}\right)^2 \int \frac{d^D P}{(2\pi)^D} \frac{i}{P^2 - M^2} \frac{i}{(P_\parallel + k)^2 - m^2} \\
&\sim \frac{\ell_D}{\Lambda^{D-6}} \frac{1}{\ell_D} \left[\Lambda^{D-4} + \Lambda^{D-6} k^2 + \cdots\right],
\end{aligned} \tag{3.18}$$

where k is a 4-momentum on the brane and $P_\parallel$ is the projection of the bulk momentum P in the direction parallel to the 3-brane. The kinematic suppression from the loop cancels the enhancement factor from the coupling, and the result is the same order of magnitude as the tree-level term.

3.3 Estimates of soft SUSY breaking

We now estimate the size of the various parameters in this theory. We first consider the size of the extra dimensions. The extra dimensions must be large enough

D	$\Lambda L_{\max}$	$e^{-\Lambda L_{\max}/2}$	$\Lambda r_{\max}$	$e^{-\Lambda r_{\max}}$
5	740	3×10^{-162}	118	4×10^{-52}
6	63	2×10^{-14}	18	2×10^{-8}
7	29	6×10^{-7}	11	3×10^{-5}
8	20	5×10^{-5}	8.7	2×10^{-4}
9	16	3×10^{-4}	8.0	3×10^{-4}
10	14	9×10^{-4}	7.8	4×10^{-4}
11	13	1×10^{-3}	7.8	4×10^{-4}

Table 1: Estimates for the toroidal compactification length L and spherical compactification radius r, as well as the exponential suppression factor for massive propagation between two branes of maximal separation.

to suppress FCNC's, but small enough so that the standard-model gauge bosons (which propagate in the bulk) have 4-dimensional gauge couplings of order 1. Using the results of the previous subsection, the maximum value D-dimensional gauge coupling is

$$g^2_{D,\max} \sim \frac{\ell_D}{\Lambda^{D-4}}. \tag{3.19}$$

Since $g_4^2 \sim g_D^2/V_{D-4} \sim 1$, this gives a maximum value for the volume of the extra dimensions. For a symmetric toroidal compactification the volume of the compact space is $V_{D-4} = L^{D-4}$, where L is the length of the sides, and we obtain

$$L_{\max} \sim \frac{\ell_D^{1/(D-4)}}{\Lambda}. \tag{3.20}$$

For a spherical compactification with radius r, we obtain

$$r_{\max} \sim \left(\frac{\ell_D}{\Omega_{D-3}}\right)^{1/(D-4)} \frac{1}{\Lambda}. \tag{3.21}$$

Numerically, $L_{\max}, r_{\max} \gtrsim 10/\Lambda$ for all $5 \le D \le 11$. This is sufficiently large to suppress FCNC's, since these are exponentially suppressed by Yukawa factors. See table 1. Note that for $D = 5$ or 6, we do not need the D-dimensional gauge coupling to be strongly coupled at the scale Λ, but strong coupling is required for larger values of D.

We now estimate the size of the soft SUSY breaking terms. We first consider the possibility that the theory is weakly coupled at the D-dimensional Planck scale M_D, and that all couplings are order 1 in units of M_D. Actually, the bulk gauge coupling must be somewhat larger than this in order that $g_4 \sim 1$, but e.g. for $D = 5$ we require only $g_5^2 \gtrsim 10/M_D$ in order to obtain $LM_D \gtrsim 10$. In this case, it is plausible that the VEV that breaks $\mathrm{U}(1)_X$ is $v \sim M_D$, and that the contact terms eqs. (1.7) and (1.8) are also order 1 in units of M_D. In this case, all soft masses are of order

$$m_{\text{soft}} \sim \frac{\langle F_\Sigma \rangle}{M_D} \sim m_{3/2} \left(V_{D-4} M_D^{D-4}\right)^{1/2}, \tag{3.22}$$

where the 4-dimensional Planck scale M_4 is given by

$$M_4^2 = V_{D-4} M_D^{D-2} \,. \tag{3.23}$$

If the compact dimensions are very large, the gravitino is the LSP, although this is easily avoided for $D = 5$ or 6.

We now consider the alternative that all couplings in the theory are strong at the fundamental scale Λ. We first estimate the value of Λ in this scenario. In this case, the gauge couplings are as large as possible so we take $r \sim r_{\rm max}$. Combining this with the estimate of Λ given in eq. (3.13) and the formula for the 4-dimensional Planck scale eq. (3.23) we obtain the simple result

$$\Lambda \sim M_4 \,. \tag{3.24}$$

We now estimate the standard-model scalar masses, gaugino masses, and μ and $B\mu$ terms. The gaugino mass and μ and $B\mu$ terms are estimated from the coefficients of the higher-dimension operators eqs. (1.7) and (1.8) connecting the bulk gauge and Higgs fields to the fields propagating on the 3-branes. Using the estimates for strongly coupled theories given in the previous subsection, we find that the effective 4-dimensional theory (written in terms of canonically normalized fields) contains the terms

$$\begin{aligned} \mathcal{L}_4 \sim & \int d^4\theta \left[\frac{1}{4\pi\Lambda} \Sigma^\dagger (H_u H_d + H_u^\dagger H_u + H_d^\dagger H_d) + \frac{1}{\Lambda^2} \Sigma^\dagger \Sigma H_u H_d \right] + \\ & + \int d^2\theta \, \frac{1}{4\pi\Lambda} \Sigma W^\alpha W_\alpha + \text{h.c.} \end{aligned} \tag{3.25}$$

Here the relation $g_4^2 \sim 1$ has been used to eliminate the dependence on the D-dimensional loop counting parameter ℓ_D. We therefore find

$$m_\lambda \sim \mu \sim \frac{1}{4\pi} m_{3/2}, \qquad m_H^2 \sim B\mu \sim m_{3/2}^2 \,. \tag{3.26}$$

It is important that these soft masses are larger than the anomaly-mediated contributions. For example, anomaly mediation gives a contribution to gaugino masses $\Delta m_\lambda / m_\lambda \sim 1/(4\pi)$.

For the scalar masses, we use the fact that the natural size for the VEV of a dimensionless wall field in a strongly-coupled theory is $\langle \hat{\phi} \rangle \sim 1$ [18]. This gives a $\mathrm{U}(1)_X$ breaking VEV of order $v \sim \Lambda/(4\pi)$, and squark and slepton masses are

$$m_{\tilde{Q},\tilde{\ell}}^2 \sim 16\pi^2 X_Q X_\Sigma m_{3/2}^2 \,, \tag{3.27}$$

where X is the $\mathrm{U}(1)_X$ gauge charge. These estimates give $m_{\tilde{Q}}/m_\lambda \sim 16\pi^2 (X_Q X_\Sigma)^{1/2}$. If we take this at face value, we must choose the $\mathrm{U}(1)_X$ charges of Q and Σ to be small in order to avoid unrealistically large squark masses.[7] However, we should

[7] Recall that the $\mathrm{U}(1)_X$ charges are normalized so that the field that obtains a VEV has $X = +1$.

allow uncertainties in the estimates of strong-interaction quantities at the level of an order of magnitude, in which case we can easily obtain scalar and gaugino masses of the same order with only moderately small $\mathrm{U}(1)_X$ charges. In addition, there is the possibility that there are other moderately large factors that modify this result. For example, large-N counting in the sector that breaks the $\mathrm{U}(1)_X$ gives $v \sim \sqrt{N}\Lambda/(4\pi)$, which reduces $m^2_{\tilde{Q}}$ by a factor of N. We expect realistic models will have a large number of degrees of freedom (e.g. $N \gtrsim 10$) that will affect the estimates above in other sectors as well.

The above estimate for the scalar masses uses the results of the tree-level calculation of section 2. It is important to know whether these results are qualitatively reliable in the strongly-coupled case we are considering. The key point is that the squark masses arise from a non-local effect in the D-dimensional theory, and are therefore insensitive to the short-distance physics. In particular, the leading contribution to the scalar masses arises from the exchange of the lightest $\mathrm{U}(1)_X$ KK mode, which has mass of order $1/r \ll \Lambda$. As long as the strong dynamics gives rise to these light states with the symmetry breaking patter assumed, we expect the estimates above to be valid at the order-of-magnitude level.

Another attractive possibility is that the couplings of fields on the 3-branes are perturbative (dimensionless couplings of order 1), while the bulk fields are strongly coupled. In this case, the estimates for the gaugino masses and μ and $B\mu$ terms are the same as in the strongly-coupled scenario discussed above. However, in this case, the natural size for the VEV that breaks $\mathrm{U}(1)_X$ is $v \sim \Lambda$, which gives $m_{\tilde{Q}}/m_\lambda \sim 4\pi(X_Q X_\Sigma)^{1/2}$.

Of course, the set-up we have described does not explain all small parameters in the low-energy effective field theory. For example the estimates above tell us that the Yukawa couplings are order 1 (even in the strongly-coupled case). This is a good starting point for a theory of flavor, since it can explain why the top Yukawa coupling is perturbative but order 1, but clearly additional structure is needed to explain why the other Yukawa couplings are suppressed. There are also other small numbers (e.g. in cosmology) that are not explained in this scenario as elaborated so far. It would be interesting to see if there are higher-dimensional mechanisms that can explain these hierarchies and small numbers in our scenario, perhaps analogous to those considered in the context of millimeter-sized extra dimensions [20].

3.4 Phenomenology

We now comment briefly on the phenomenology of these models.

The first important point is that the scalar masses generated by $\mathrm{U}(1)_X$ exchange are naturally flavor-diagonal if $\mathrm{U}(1)_X$ is broken in the hidden sector and if $\mathrm{U}(1)_X$ commutes with flavor symmetries. This is perhaps the most attractive feature of the present class of models.

If the Yukawa couplings arise from $\mathrm{U}(1)_X$ invariant effects, the fact that the down-type quarks and the leptons get masses from the same Higgs field implies the scalar mass relation

$$m^2_{\tilde{L}L} + m^2_{\tilde{e}R} = m^2_{\tilde{Q}L} + m^2_{\tilde{d}R}\,, \tag{3.28}$$

up to small radiative corrections.[8] This is the same as a SU(5) GUT relation, but in the present models it may hold even in the absence of grand unification (e.g.in string unification).

Another general feature of the present models is that A terms can naturally be small. In conventional hidden sector models these are generated by operators of the form

$$\Delta\mathcal{L}_{\text{eff}} \sim \int d^2\theta\, \frac{1}{M} \Sigma Q Q H_{u,d} + \text{h.c.}\,, \tag{3.29}$$

but in the present models these are exponentially suppressed since the hidden and visible sectors are separated. A terms can arise from the operator $\int d^4\theta (\Sigma^\dagger X \Sigma)(Q^\dagger X Q)$ in models where $\langle\Sigma\rangle \neq 0$ in addition to $\langle F_\Sigma\rangle \neq 0$. In such models, we expect $\langle\Sigma\rangle \ll M_D$, so this contribution is also suppressed. There is an unavoidable contribution to the A terms from anomaly mediation [7], but it is suppressed both by loop factors and Yukawa couplings, and is therefore negligible for most purposes.

Another possibility is that the μ and $B\mu$ terms are generated by the VEV of a singlet S in the visible sector. Models of this type require the following superpotential terms in the effective 4-dimensional theory:

$$W_{\text{obs}} = \lambda S H_u H_d + \frac{\kappa}{3} S^3 + \text{Yukawa couplings}\,. \tag{3.30}$$

The terms involving S are not $\mathrm{U}(1)_X$ invariant, but they can arise with couplings of order 1 if the Higgs propagates in the bulk and S propagates either in the bulk or on the hidden-sector brane. In that case, these terms can arise from higher-dimension brane superpotential terms involving the field that breaks $\mathrm{U}(1)_X$.

One very attractive feature of this model is that the absence of large A terms implies that it solves the SUSY CP problem. In this model, the only terms in the effective lagrangian with possible CP-violating phases are the visible sector superpotential couplings and the hidden-sector superpotential term that gives rise to gaugino masses

$$W_{\text{hid}} = \frac{c}{M_D} \Sigma W^\alpha W_\alpha\,. \tag{3.31}$$

(Here we assume that the theory is embedded in a GUT so that there is only one independent gaugino mass.) The phases in λ, κ, and c can be rotated away as follows. A $\mathrm{U}(1)_R$ rotation (where all matter fields have $r = +2/3$) can be used to make the gaugino mass real. Then S can be rephased to make κ real. Finally, we

[8]The radiative corrections may be significant for large $\tan\beta$.

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use a $\mathrm{U}(1)_{\mathrm{PQ}}$ rotation to make λ real, where the PQ charge of all quark and lepton fields is $-1/2$, $H_{u,d}$ have charge $+1$, and S has vanishing PQ charge. Note that these transformations will not eliminate phases in the A terms in general, but we have seen that it is natural for the A terms to arise only from anomaly mediation. This means that they are loop suppressed, and also their phases come from the phases in the superpotential couplings.

If the μ problem is solved by the Giudice-Masiero mechanism, there are uncontrolled phases in the gaugino mass and μ and $B\mu$ terms that cannot be eliminated by field redefinitions. These models therefore have a SUSY CP problem identical to conventional hidden sector models.

In many ways the phenomenology of these models is very similar to conventional hidden sector models: scalar and gaugino masses are of order $m_{3/2}$, and in the context of GUT models, scalar masses in the same GUT multiplet and gaugino masses unify at the GUT scale. In fact, since the scalar mass-squared terms are controlled by $\mathrm{U}(1)_X$ gauge charges, it is natural for them to be equal (or have simple rational ratios) at the $\mathrm{U}(1)_X$ breaking scale even in the absence of grand unification (e.g.in string unification). This may also occur in 'anomalous U(1)' models [21]. This gives the possibility of a rather distinctive signature, namely that scalar masses unify while gaugino masses do not.

FCNC's are exponentially suppressed in this model, and are therefore unobservably small unless the value radius accidentally puts them near the experimental limits.

Finally, we mention that the $D-4$ 'extra' polarizations of the bulk $\mathrm{U}(1)_X$ gauge field (A^I, $I = 4, \ldots, D-1$) do not get mass from the Higgs mechanism on a 3-brane, and therefore may be light. The same may be true for other components of the supersymmetric gauge multiplet (e.g. gauginos). In the specific model constructed in section 3.1 these states obtain masses of order the compactification scale by the orbifold projection. However, it may be interesting to consider other scenarios where these fields are light. By gauge invariance, the fields A^I can have non-derivative couplings only to charged bulk fields. If there are no charged bulk fields, these fields are derivatively coupled (through the $\mathrm{U}(1)_X$ gauge field strength) with higher-dimension operators suppressed by powers of M_D. Such fields are not visible in terrestrial experiments and will not be in equilibrium in the early universe provided that the inflationary reheat temperature is smaller than M_D. If the Higgs fields propagate in the bulk and are charged under $\mathrm{U}(1)_X$, there is an important coupling from the Higgs kinetic term $D^M H^\dagger D_M H = H^\dagger H A^I A_I + \cdots$. In fact, this interaction gives the fields A^I a (positive) weak-scale mass. This scenario is very constrained, especially since we expect that supersymmetric partners of the A^I will also be light. Analogous remarks are expected to hold for supersymmetric partners of the A^I fields. Since these possibilities are highly model-dependent, we will not analyze them further here.

4. Conclusions

We have argued that a bulk $\mathrm{U}(1)_X$ gauge field broken on the hidden-sector 3-brane is an attractive candidate for the messenger of supersymmetry breaking. This scenario automatically suppresses flavor-changing neutral currents independently of the flavor structure at the fundamental Planck scale, while at the same time naturally giving positive scalar mass-squared terms. Gaugino masses are naturally generated if the standard-model gauge fields propagate in the bulk. The μ problem can be solved either by the Giudice-Masiero mechanism if the Higgs fields propagate in the bulk, or by the VEV of a field in the visible sector. In the latter case, the SUSY CP problem is automatically solved because of the absence of large A terms.

In these models, the scales of the supersymmetry breaking parameters can be naturally related if we assume that all microscopic parameters are order 1 at the fundamental Planck scale. Another natural possibility is that all interactions in the theory are strongly-coupled at a fundamental scale near the D-dimensional Planck scale. In this scenario, the only large hierarchy is the 'large' size of the extra dimensions, which need only be an order of magnitude compared to the fundamental scale. This is attractive from the point of view of string theory, where there are severe difficulties in formulating realistic theories at weak coupling.

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